A MACHINE SPEED OPTIMIZATION PROBLEM IN A CAPACITATED FELT PRODUCTION SYSTEM

Banu KABAKULAK[1*]

[1] Department of Industrial Engineering, İstanbul Bilgi University, Türkiye,
ORCID No : https://orcid.org/0000-0001-8613-3266

Keywords
Felt production
Lot sizing
Machine speed
Mixed-integer programming

Abstract
In this study, we analyze a felt production system with unique requirements, such as maintaining machine speeds within specific limits to facilitate successful chemical reactions. By incorporating machine speed constraints and restrictions on both work-in-process (WIP) and end-product inventories, we aim to determine the optimal production quantities for each felt type and the corresponding machine speeds over a defined planning horizon. The objective is to minimize total costs, including machine setup and production costs as well as WIP and end-product inventory holding costs. To achieve this, we introduce the Machine Speed Optimization (MSO) problem and adapt it to the specific requirements of a felt manufacturing company operating in Istanbul, Türkiye. The MSO problem is formulated as a mixed-integer linear programming (MILP) model, which is NP-hard to solve for optimal production decisions. We validate the MSO model using the felt manufacturing company's case over five periods, demonstrating its effectiveness in automating production planning and machine speed decisions. The simulations for a 5-day planning horizon demonstrate a cost reduction of 3853 TL, a 24% decrease in WIP inventory, and up to a 15% improvement in machine utilization compared to the current practices of the felt manufacturing company. Additionally, the optimized machine speeds achieved through the MSO model enable the system to increase throughput by 11%. Experimental analysis of computational complexity reveals that the MSO model can generate an optimal 6-month production plan, including machine speeds, in under one hour.

KAPASİTELİ BİR KEÇE ÜRETİM SİSTEMİNDE MAKİNE HIZI EN İYİLEME PROBLEMİ

Anahtar Kelimeler
Keçe üretimi
Parti Büyüklüğü
Makine hızı
Karma-tamsayılı programlama

Öz
Bu çalışmada, kimyasal reaksiyonların başarılı bir şekilde gerçekleşmesini sağlamak için makine hızlarının belirli sınırlar içinde tutulması gibi kendine özgü gereksinimlere sahip bir keçe üretim sistemi ele alınmıştır. Makine hızı kısıtları ve hem yarı mamul (YM) hem de nihai ürün stok sınırlamaları dikkate alınarak, her keçe türü için en iyi üretim miktarlarının ve ilgili makine hızlarının belirlenmesi amaçlanmaktadır. Amaç, makine kurulum ve üretim maliyetleri ile YM ve nihai ürün stok tutma maliyetlerini en küçüklemektir. Bu doğrultuda, Makine Hızı En İyileme (MHE) problemi tanımlanmış ve İstanbul, Türkiye'de faaliyet gösteren bir keçe üretim firmasının gereksinimlerine uyarlanmıştır. MHE problemi, en iyi üretim kararlarının bulunmasının NP-zor olduğu karma-tamsayılı doğrusal programlama modeli olarak ifade edilmiştir. Önerilen MHE modelinin üretim planlaması ve makine hız kararlarının otomasyonu konusunda etkinliği keçe üretim firmasının beş dönemlik verileri üzerinde doğrulanmıştır. 5 günlük planlama ufku için yapılan simülasyonlar, firmanın mevcut uygulamasına kıyasla toplam maliyette 3853 TL azalma, YM stoklarında %24 düşüş ve makine kullanım oranında %15'e varan iyileşme sağlandığını göstermektedir. Ayrıca, MHE modeli ile en iyilenen makine hızları, sistemin üretim kapasitesini %11 arttırmıştır. MHE modelinin karmaşıklığına dair deneysel analizler, modelin makine hızlarını içeren 6 aylık bir üretim planını bir saatten daha kısa sürede en iyileyebildiğini ortaya koymuştur.

Araştırma Makalesi		Research Article	
Başvuru Tarihi	: 01.04.2025	Submission Date	: 01.04.2025
Kabul Tarihi	: GG.AA.YYYY	Accepted Date	: GG.AA.YYYY

* Sorumlu yazar: eposta@aaa.edu.tr
https://doi.org/10.xxxxx/ogummf.10xxxxx

1. Introduction

The Lot Sizing (LS) problem focuses on determining the optimal production quantities of items over a planning horizon to minimize setup, production, and inventory holding costs (Karimi, Ghomi and Wilson, 2003; Quadt, 2004). Identifying the optimal production quantities enables informed decision-making to enhance system performance and productivity, thereby boosting the company's competitiveness in the market.

In a production system, the end-product can consist of discrete items, such as metal parts produced by a mold, or non-discrete items, such as float glass or felt, where raw materials flow continuously through machines (Özdamar and Bozyel, 2000; Özyörük and Erol, 2000). When resources like machine time, manpower, inventory, or budget are constrained, the LS problem becomes a Capacitated LS Problem (CLSP) (Ganesh, 2019). While the single-item uncapacitated LS problem can be solved in polynomial time (Wagner and Whitin, 1958), the single-item CLSP is classified as NP-hard (Florian, Lenstra and Kan, 1980).

Depending on the application, the LS problem may have various extensions, such as incorporating machine setups for each product type, allowing early-period demand fulfillment through backlogging, imposing time restrictions on demand (time windows), or enabling simultaneous production of multiple items in a single production run (co-production), among others (Jans and Degraeve, 2008; Kimms, 2012).

Koca, Yaman and Aktürk (2015) addressed a CLSP with stochastic demands and controllable processing times. The authors formulated the problem as a non-linear Mixed-Integer Programming (MIP) model to optimize machine speeds and minimize overall production costs. Since the problem is NP-Hard, the authors strengthened their formulation using second-order cone programming to obtain exact solutions within acceptable computation times. Their findings indicated that the most significant cost improvements occurred in instances with high machine setup costs and medium capacity levels. However, their approach was limited to a capacitated single-machine, single-product system.

Akbalik, Penz and Rapine (2015) explored another CLSP extension by introducing inventory bounds. They demonstrated that a single-item problem with stationary production capacity, time-dependent inventory bounds, and concave costs could be solved to optimality in polynomial time. However, for the multi-item version, the problem becomes strongly NP-hard.

The literature includes several studies addressing specific implementations of the LS problem across various industries (Ramya, Rajendran, Ziegler, Mohapatra and Ganesh, 2019). For example, Martinez, Toso and Morabito (2016) investigated the LS problem in the molded pulp packaging industry to determine optimal molding patterns for the molding machine alongside lot sizing and scheduling decisions. They developed a MIP model to minimize total setup and inventory costs while accounting additional technical constraints. Despite the problem is NP-hard, the authors did not propose any heuristic approaches for their production planning and process selection model.

In the furniture industry, wooden parts need to be cut for assembling the final products. In this context, Gramani, França and Arenales (2009) integrated the cutting stock and production planning processes into a single mathematical model. They proposed a heuristic method based on Lagrangian relaxation, which allowed them to handle the lot-sizing and cutting stock subproblems separately, achieving computational efficiency.

Remanufacturing worn-out products to restore them to a like-new condition is common in various industries, including single-use cameras, car engines, and automotive service parts. Naeem, Dias, Tibrewal, Chang and Tiwari (2012) considered a single-item dynamic LS problem with both manufacturing and remanufacturing options. Their model allowed for backlogging, *i.e.,* the demands of earlier periods could be satisfied later. The objective was to minimize total production, holding, and backlog costs. The solution employed dynamic programming, which could handle both deterministic and stochastic demand patterns, variable manufacturing unit costs, and backlogging. The flexibility of their approach is a notable advantage of their solution method.

In third-party warehousing systems, supply contracts often allow customer demand to be fulfilled within a specified time window without incurring penalties (Chung-Yee, Çetinkaya and Wagelmans, 2000). The authors demonstrated that extending the single-item LS model to include time windows can be solved in polynomial time using a dynamic programming approach.

On the other side, Brahimi, Dauzere-Peres and Najid (2005) noted that the multi-item CLSP with time windows is strongly NP-hard. To address this complexity, they proposed alternative mathematical formulations and developed Lagrangian heuristics based on relaxing various constraint sets. Among these, the best heuristic performance was achieved by relaxing only the capacity constraints. Additionally, Brahimi et al. (2005) enhanced the solution quality further by integrating valid inequalities into their formulations.

Wu, Shi and Duffie (2010) addressed the multi-item CLSP with product-dependent setup times on machines. Their approach combined a column generation algorithm with a relax-and-fix heuristic to tackle the problem efficiently. Similarly, Yanzhi, Yi and

Wang (2012) investigated the same problem and proposed a three-stage solution method. In the first stage, they preprocessed the instance to account for joint setup costs. The second stage constructed an initial feasible solution using a period-by-period heuristic. Finally, in the third stage, they refined the solution by solving a series of subproblems, further enhancing the overall result.

Absi, Detienne and Dauzere-Peres (2013) studied the CLSP with product-dependent setup times and lost sales. They approached the problem by relaxing capacity constraints and solving single-item uncapacitated LS problems with lost sales. To ensure capacity constraints were met, they employed a non-myopic heuristic strategy to refine the subproblem solutions and align them with the machine capacity limits. In a related extension, Ben Ammar, Avadi and Masmoudi (2020) addressed a multi-item CLSP with setup times and backlogging, allowing demands from earlier periods to be fulfilled later. The authors developed a multi-objective particle swarm optimization heuristic designed to simultaneously minimize total costs and total inventory levels, providing an effective approach for balancing competing objectives in such systems.

Designing sustainable production systems by considering energy consumption in job shops is of critical importance. In this context, Masmoudi, Yalaoui, Quazene and Chehade (2016) investigated the multi-item CLSP with an added focus on minimizing energy consumption. The energy consumption was modeled as a function of the power requirements of the machines. They formulated the problem as a MIP model and proposed a fix-and-relax heuristic method to efficiently solve the problem, balancing production costs with energy usage.

Shabtay and Steiner (2007) provided a comprehensive review of the literature on scheduling problems involving controllable processing times. Building on this area, Geng and Yuan (2023) examined the single-machine multiple-project scheduling problem with controllable processing times. Their objective was to minimize the total cost of altering machine speeds while reducing the weighted number of tardy jobs in the schedule. They demonstrated that the problem is NP-hard and proposed a dynamic programming (DP) algorithm to solve it.

Similarly, Levin and Shusterman (2023) studied single-machine preemptive scheduling with continuous controllable processing times. Their goal was to maximize the total reward for early task completion by adjusting machine speeds dynamically. They developed a pseudo-polynomial DP algorithm, which was further refined into a Fully Polynomial Time Approximation Scheme (FPTAS). However, these models are limited to single-machine scenarios and need to be extended to multi-machine environments to address more complex and practical scheduling challenges.

Wang, Wu, Chu and Yu (2022) addressed flexible machine speeds in an unrelated parallel machine environment with machine- and sequence-dependent setup costs. Their objective was to maximize the difference between the total realized state of all jobs and the makespan, using a logic-based Benders decomposition approach for their MIP model. However, their method is specifically tailored to unrelated parallel machines and is not applicable to items that follow different production sequences across multiple machines.

In flow shops, controlling processing times can significantly impact energy consumption. Renna (2023) proposed two heuristic control policies, centralized and distributed, to manage processing times in a flow shop under limited budget and energy constraints. The distributed policy relied on a multi-agent architecture, where stations exchanged their current state information with neighboring stations at each period. Conversely, the centralized policy provided more efficient delivery of items with fewer time delays compared to the distributed policy, as expected. While both approaches showed promise, being heuristic methods, they still offer significant room for improvement in efficiency and effectiveness.

This study contributes to the literature by introducing an extension of the finite horizon LS problem specifically designed for a felt production system, integrating machine capacity constraints and inventory bounds on both end-products and WIP. In the felt production process, machines can handle semi-products uniformly, irrespective of their final product types, eliminating product-dependent setup costs. Additionally, backlogging is prohibited to ensure on-time delivery of customer demands.

To promote sustainability, we extend the traditional LS problem to the Machine Speed Optimization (MSO) problem by incorporating machine speed-dependent operating costs, thereby minimizing energy consumption as part of the production planning process. Since energy usage correlates directly with machine speeds, we optimize execution speeds for each machine across all planning periods. The proposed MSO model contributes to the literature by addressing the simultaneous challenges of managing a continuous-flow, multi-product production system on capacitated multi-machines within an inventory-constrained environment, while also optimizing machine speeds for energy efficiency. This dual focus on production planning and sustainability makes the MSO model a novel approach for felt manufacturing systems.

This paper can be summarized as follows:

- We address a capacitated multi-item MSO problem in job shop systems, where machine speeds can be adjusted to reduce the overall production cost.
- Specifically, we examine a felt production system where products continuously flow according to their production routes across multiple machines.
- For this capacitated felt production environment, we propose a MILP formulation to optimally control the processing times while minimizing the total production cost. This model extends the CLSP (see Section 3).
- We adapt our MSO model to a felt production company operating in İstanbul, Türkiye, as a case study (see Section 4).
- We conduct computational experiments on this case study to demonstrate the effectiveness of sustainable production control. Furthermore, we experimentally demonstrate the computational complexity of the MSO model using different demand scenarios and planning horizon parameters (see Section 5).

In the rest of the paper, we formally define the MSO problem for a felt production system in Section 2, develop a MILP formulation for the MSO problem in Section 3, introduce a case study based on an existing felt manufacturing company in Section 4, provide the experimental results obtained from the case study in Section 5. Section 6 concludes the paper with some remarks and comments on the future research tracks.

2. Problem Definition

In this section, we present the machine speed optimization problem, an extension of CLSP, specifically designed for a felt production system.

Felt is a textile material produced by matting fibers, such as wool or synthetic fibers, through a process known as felting. The production begins with a cleaning step, where the fibers are washed, followed by a carding step, where fibers are aligned using combs on a carding machine to ensure uniform felt production.

In the laying step, the fibers are arranged in multiple layers to form a web, which is then moistened with water in the wetting step to promote bonding. Depending on the product type, the wetting step may also involve the application of specific chemicals or dyes to achieve desired properties.

In the felting step, the wet fiber web is subjected to pressure, facilitating the interlocking of fibers to form a cohesive material. This is followed by the rinsing and drying steps, where the felt is thoroughly rinsed to eliminate residual soap or chemicals and then dried to achieve the desired texture. Finally, in the cutting step, a cutting machine trims the felt into specific shapes or sizes, completing the production process.

In a felt production system, there are I products, represented by the set $\mathcal{I} = \{1, \dots, \mathrm{I}\}$, as given in Table 1. These products flow through M machines, denoted by the set $\mathcal{M} = \{1, \dots, M\}$. The MSO problem involves determining the production quantities for each product $i \in \mathcal{I}$ over a planning horizon of T periods, where the planning periods are represented by the set $\mathcal{T} = \{1, \dots, T\}$. The flow of products through the machines is captured by the production sequence matrix $\boldsymbol{A}$, a three-dimensional binary matrix with dimensions $I \times (M+1) \times (M+1)$. The matrix $\boldsymbol{A}$ indicates the flow sequence of each product. Specifically, if a product i moves to machine m_2 after machine m_1, then $\boldsymbol{A}_{im_1m_2} = 1$, and 0 otherwise. The dimensions of $\boldsymbol{A}$ account for a dummy initial machine (denoted as machine−0) and a dummy final machine (denoted as machine−$(M+1)$), included for all products $i \in \mathcal{I}$. Hence, $m_1 \in \{0, \dots, M\}$ and $m_2 \in \{1, \dots, (M+1)\}$. Note that, $\boldsymbol{A}_{i0m} = 1$ if m is the first machine and $\boldsymbol{A}_{im(M+1)} = 1$ if m is the last machine in the production sequence of product i.

Felt production has distinct characteristics, such as the continuous nature of production, measured in meters rather than discrete units. Additionally, certain production stages, such as chemicalization, impose specific constraints on machine speed, requiring it to remain within predefined bounds to ensure successful production. The minimum and maximum production capacities of a machine m are denoted as C_m^{min} and C_m^{max}, respectively. Furthermore, the facility operates for C_t^{fac} minutes during a given period t.

Table 1. List of Parameters

$\mathcal{I}$	Set of products
I	Number of products
$\mathcal{M}$	Set of machines
M	Number of machines
$\mathcal{T}$	Set of periods
T	Number of periods (planning horizon)
$\boldsymbol{A}$	Production sequence matrix
C_m^{max}	Maximum production capacity of machine m per period (meters/period)
C_t^{fac}	Plant capacity in period t (mins/period)
d_{it}	Demand of product i in period t (meters)
u_m^{max}	WIP inventory capacity of machine m

	(meters)
s^{max}	End-product inventory capacity (meters)
r_{mt}	Operating cost of machine m in period t (TL/meter)
f_{mt}	Fixed setup cost of machine m in period t (TL/period)
p_{it}	Production cost of product i in period t (TL/meter)
h_{it}	End-product inventory holding cost of product i in period t (TL/meter)
w_{imt}	WIP holding cost of product i on machine m in period t (TL/meter)

The demand for a product i in a period t is d_{it} meters. The end-products are stored in a warehouse with a maximum storage capacity of s^{max} meters. A machine m has a WIP inventory capacity of u_m^{max} meters.

The continuous production of felt in meters is directly influenced by the operating cycles and speeds of the machines. A fixed setup cost f_{mt} incurs if a machine m operates in a period t. Additionally, r_{mt} is the operating cost associated with the unit increment on the speed of machine m in a period t. The production cost of a product i in a period t is p_{it} TL/meter. The WIP holding cost of product i on machine m in period t is w_{imt}, and the end-product inventory holding cost of product i in period t is given by h_{it}.

The MSO problem introduced in this work extends the well-known LS problem by accounting for the continuous production of multiple products, each following distinct machine routes based on their specific processing requirements. The corresponding felt production system has machine capacities, WIP inventory limits, and end-product storage constraints. Additionally, the operational costs are dependent on the machine speeds, which further complicates the optimization process. The objective of the MSO problem is to minimize the total operational and production costs as well as the inventory holding costs.

The main assumptions of the model can be summarized as follows:

- The planning horizon is divided into periods of equal length.
- Multiple products are produced, each following different production sequences in the job shop system.
- The product demands are deterministic and must be satisfied in each period.
- There are bounds on the WIP and end-item inventories.
- The machine speeds are subject to both lower and upper bounds.
- Each machine has a limited production capacity.
- Machines incur both fixed setup costs and variable operating costs depending on their speed.

We formally introduce the generic MSO model in Section 3. In Section 4.1, we describe the felt manufacturing company case. Section 4.2 focuses on adapting the MSO model to the specific needs of the case facility.

3. Mathematical Formulation

In this section, we formulate the MSO problem for a felt production system, as defined in Section 2, as a MILP model. Section 4 presents the implementation of the MSO model for the felt manufacturing company case. This research is in accordance with the Research and Publication Ethics.

Table 2 summarizes the decision variables used to determine the optimal production quantities and machine execution speeds in a felt production job shop. In particular, the binary variable x_{mt} is one if machine m is operating in period t, and zero otherwise. If a machine m is operational, then it produces y_{imt} meters of product i in period t. The production level C_{mt} (measured in meters) represents the total amount of production produced by machine m in period t. It is determined by the machine's operating speed and the duration of operation during that period, considering any capacity limits. The corresponding machine speed v_{mt} in period t can be calculated as $v_{mt} = C_{mt}/C_t^{fac}$ meters/min, where C_t^{fac} is the facility capacity. In period t, the end-product inventory level of product i is s_{it} meters and the accumulated WIP inventory of product i in front of machine m is u_{imt} meters.

Table 2. List of Decision Variables

x_{mt}	1 if machine m is operating in period t 0 otherwise
y_{imt}	Amount of production of product i on machine m in period t (meters)
C_{mt}	Production level of machine m in period t (meters/period)
u_{imt}	WIP inventory level of product i in front of machine m in period t (meters)
s_{it}	End-product inventory level of product i in period t (meters)

The Machine Speed Optimization (MSO) Model:

$$\min \sum_{m\in\mathcal{M},t\in\mathcal{T}} f_{mt}x_{mt} + \sum_{i\in\mathcal{I},m\in\mathcal{M},t\in\mathcal{T}} \boldsymbol{A}_{im(M+1)}\, p_{it}y_{imt} + \sum_{i\in\mathcal{I},t\in\mathcal{T}} \left[\left(\sum_{m\in\mathcal{M}} w_{imt}u_{imt}\right) + h_{it}s_{it}\right] + \sum_{m\in\mathcal{M},t\in\mathcal{T}} r_{mt}\, C_{mt} \tag{1}$$

s.t.

$$\sum_{i\in\mathcal{I}} y_{imt} = C_{mt} \quad \forall m\in\mathcal{M}, t\in\mathcal{T} \tag{2}$$

$$\sum_{t\in\mathcal{T}} y_{imt} \le \left(\sum_{t\in\mathcal{T}} d_{it}\right)\left(\sum_{m'\in\mathcal{M}\cup\{M+1\}} \boldsymbol{A}_{imm'}\right) \quad \forall i\in\mathcal{I}, m\in\mathcal{M} \tag{3}$$

$$\sum_{m_1\in\mathcal{M}} \boldsymbol{A}_{im_1m_2}y_{im_1(t-1)} + u_{im_2(t-1)} = y_{im_2t} + u_{im_2t} \quad \forall i\in\mathcal{I}, t\in\mathcal{T},\, m_2\in\mathcal{M} \tag{4}$$

$$u_{im(t-1)} = y_{imt} + u_{imt} \quad \forall i\in\mathcal{I}, t\in\mathcal{T}, m\in\mathcal{M} \text{ with } \boldsymbol{A}_{i0m} = 1 \tag{5}$$

$$u_{im0} = \left(\sum_{t\in\mathcal{T}} d_{it}\right)\boldsymbol{A}_{i0m} \quad \forall i\in\mathcal{I}, m\in\mathcal{M} \tag{6}$$

$$\sum_{m\in\mathcal{M}} \boldsymbol{A}_{im(M+1)}\, y_{imt} + s_{i(t-1)} = d_{it} + s_{it} \quad \forall i\in\mathcal{I}, t\in\mathcal{T} \tag{7}$$

$$s_{i0} = 0 \quad \forall i\in\mathcal{I} \tag{8}$$

$$C_{mt} \le C_m^{max}x_{mt} \quad \forall m\in\mathcal{M}, t\in\mathcal{T} \tag{9}$$

$$\sum_{i\in\mathcal{I}} u_{imt} \le u_m^{max} \quad \forall m\in\mathcal{M}, t\in\mathcal{T} \tag{10}$$

$$\sum_{i\in\mathcal{I}} s_{it} \le s^{max} \quad \forall t\in\mathcal{T} \tag{11}$$

$$y_{im0} = 0 \quad \forall i\in\mathcal{I}, m\in\mathcal{M} \tag{12}$$

$$y_{imt}, C_{mt} \ge 0 \quad \forall i\in\mathcal{I}, m\in\mathcal{M}, t\in\mathcal{T} \tag{13}$$

$$u_{imt}, s_{it} \ge 0 \quad \forall i\in\mathcal{I}, m\in\mathcal{M}, t\in\mathcal{T} \tag{14}$$

$$\mathrm{x}_{mt} \in \{0,1\} \quad \forall m\in\mathcal{M}, t\in\mathcal{T} \tag{15}$$

In the MSO model, the objective (1) aims to minimize the total production costs, WIP and end-product inventory holding costs and capacity-dependent machine setup costs. Constraints (2) define the production level C_{mt} of machine m as the total production quantity in period t. Constraints (3) establish the feasibility condition for production y_{imt} by ensuring that machine m is included in the production sequence of product i.

Each machine has a WIP inventory positioned in front of it. In the production sequence of product i, let machine m_2 follow machine m_1, *i.e.*, $A_{im_1m_2} = 1$. In this case, the WIP inventory at machine m_2 is replenished by the output of machine m_1 and consumed during production at machine m_2, as expressed in the WIP flow balance constraints (4). If machine m is the first in the production sequence of product i, *i.e.*, $A_{i0m} = 1$, then the initial WIP flow balance is defined by constraints (5). For this initial machine m, the starting WIP is assumed to fully meet the demand for product i as specified in constraints (6).

The end-product inventory s_{it} is replenished by the production from the last machine m in the production sequence of product i. *i.e.*, $A_{im(M+1)} = 1$, and is reduced by the demand d_{it}. Constraints (7) define the end-product inventory balance, while constraints (8) ensure that the initial inventory is zero at the start of the planning horizon.

Due to the production system requirements and the technical limitations of machine m, there is an upper bound on the machine capacity as in constraints (9). The capacity bound is set to zero if machine m is not operational in period t, *i.e.*, $x_{mt} = 0$. Constraints (10) and (11) establish upper bounds on the WIP inventory and end-product inventory, respectively. Constraints (12) imply that there is no production occurs in the dummy period−0. Finally, constraints (13)−(15) enforce non-negativity and binary conditions on the decision variables.

In the MSO model, the binary decisions regarding machine operating periods dictate the production quantities in each period. Production in the job shop is driven by product demand and is constrained by the WIP and end-item inventory limits. The MSO model is a mixed-integer linear programming formulation and is classified as NP-Hard due to the combinatorial complexity associated with scheduling machine production.

4. Case Study: A Felt Production Company

In this section, we demonstrate the implementation of the MSO model on an existing felt manufacturing system for a company based in İstanbul, Türkiye.

4.1. The Case Overview

In our case study, we consider a textile company based in İstanbul, Türkiye, that has been producing and exporting felts for car seats since 1939. The company operates $M = 3$ different felt production machines, namely Production Line 1 (PL1, $m = 1$), Production Line 2 (PL2, $m = 2$) and Cutting Machine (CM, $m = 3$) as depicted in Figure 1. The set $\mathcal{M} = \{1,2,3\}$ collects the machine indices.

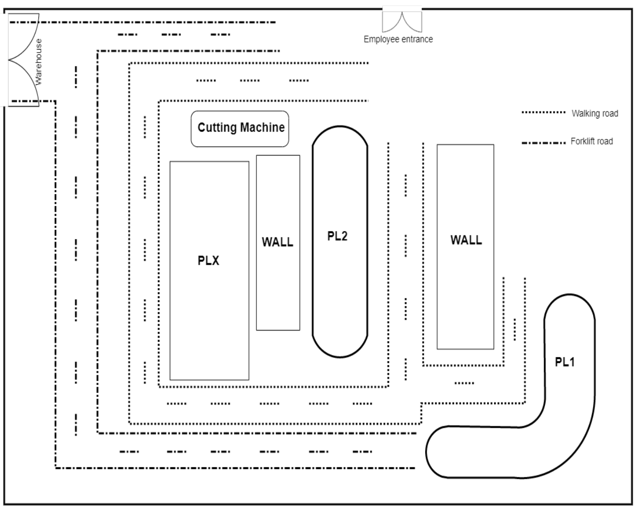

Figure 1. Facility Layout of the Felt Manufacturing Company

PL1 is responsible for three key operations: mixing, carding and needling. These mechanical operations transform the raw materials into compact felt products. The output of PL1 is a rolled cylinder of felt. In PL2, compact felt products undergo a chemical process, which includes chemicalization and drying using a conveyor dryer machine. Similar to PL1, the chemicalized felt is rolled into a cylinder at the end of PL2. CM then cuts the cylindrical felt products into plaque shapes. The production line PLX, located separately, handles a different process unrelated to the felt production. The warehouse, which is located at the top left corner of the layout, stores the end-item inventory and any excess WIP inventory.

There are $I = 4$ different types of products based on their production line and machine routing, namely non-chemical cylinder ($i = 1$), non-chemical plaque ($i = 2$), chemical cylinder ($i = 3$) and chemical plaque ($i = 4$). The product indices are listed in the set $\mathcal{I} = \{1,2,3,4\}$.

If the raw material follows only the PL1 process, then the product is a non-chemical cylinder. If the PL1 output is cut into plaque shapes in the CM, then the product is a non-chemical plaque. The output of PL1 can also be further processed in PL2 by adding some chemicals to the felt, resulting in a chemical cylinder. A chemical plaque can be produced by cutting the PL2 outputs on the CM. Thus, the production sequence matrix $\boldsymbol{A}$ can be represented as shown in Figure 2. In this matrix, $m_1 = 0$ denotes the dummy initial machine, and $m_2 = 4$ represents the dummy final machine in the production route.

A =

i	$m_1 \setminus m_2$	1	2	3	4	i	$m_1 \setminus m_2$	1	2	3	4
1	0	1	0	0	0	3	0	1	0	0	0
	1	0	0	0	1		1	0	1	0	0
	2	0	0	0	0		2	0	0	0	1
	3	0	0	0	0		3	0	0	0	0
2	0	1	0	0	0	4	0	1	0	0	0
	1	0	0	1	0		1	0	1	0	0
	2	0	0	0	0		2	0	0	1	0
	3	0	0	0	1		3	0	0	0	1

Figure 2. The Production Sequence Matrix

The company decides on the daily production schedules, with a facility capacity $C_t^{fac} = 720$ mins per day. The planning horizon of the company is set to a maximum of $T = 30$ days. The production cost p_{it} for each product i is assumed to be constant over time and is given by $p_{1t} = 6, p_{2t} = 8.5, p_{3t} = 19.5$ and $p_{4t} = 22$ TL/meter.

The end-product inventory holding cost h_{it} for each product i is time-invariant and given by $h_{1\mathrm{t}} = 0.75, h_{2\mathrm{t}} = 0.375, h_{3\mathrm{t}} = 0.75$ and $h_{4\mathrm{t}} = 0.375$ TL/meter. The cylinder products ($i \in \{1,3\}$) have higher holding cost since they require more manpower to store in the warehouse. All product types are assumed to occupy the same volume in the inventory. In the system, WIP inventory is only allowed for plaque products ($i \in \{2,4\}$) on the machines $m \in \mathcal{M}$. The WIP holding cost w_{imt} is machine- and product-dependent and can be given as $w_{32t} = w_{42t} = 11.4$ TL/meter, $w_{23t} = w_{43t} = 1.25$ TL/meter. Since, there is no empty space in front of PL2, the WIP inventory is carried to the warehouse and brought back when required. Hence,

w_{i2t} cost is higher compared to the other machines. For all products, PL1 is the first machine in the production sequence, and we assume that $w_{i1t} = 1.25$ TL/meter for all $i \in \mathcal{I}$. To eliminate the WIP inventory for products $i \in \{1,3\}$, we introduce WIP inventory constraints in the MSO model (see Section 4.2).

To meet the chemical cooling rate requirements, the machine speed of PL2 should be maintained within a specific range. Additionally, the company uses an embedded crane system that directly feeds PL2 with the outputs from PL1. However, the machine speed difference between PL1 and PL2 does not allow a smooth production. As a result, when PL2 finishes processing the outputs from PL1, an operator retrieves the required WIP inventory from the warehouse and feeds it into PL2 to continue the production process without interruption.

The WIP inventory is allowed to accumulate in front of the CM ($m = 3$) which operates at a constant speed all the time. The machine speed is measured in meters/period and a unit increase in the machine speed of PL1 results in an operating cost of $r_{1t} = 1.16$ TL, while for PL2, the cost is $r_{2t} = 3.09$ TL. CM has no speed adjustments and, therefore, no additional operating costs ($r_{3t} = 0$). These operating cost estimations account for electricity consumption and potential machine downtimes. Higher machine speeds increase the likelihood of machine failures, leading to higher operational risks. The hourly salary of a machine operator is 125 TL in the facility. An operator requires 10 mins to set up PL1, 2 mins for PL2 and 10 mins for CM. Consequently, the fixed setup costs for the machines are calculated as $f_{1t} = 20.83$ TL, $f_{2t} = 4.17$ TL and $f_{3t} = 20.83$ TL, respectively.

According to the system restrictions and technical capabilities of the machines, the machine speed ranges can be given as $v_{1t} \in [5,8]$ meters/min for PL1, $v_{2t} \in [15,18]$ meters/min for PL2 and $v_{3t} = 5$ meters/min for CM. The machine processing times are independent of the product type. Then, the maximum production capacities of the machines can be given as $C_1^{max} = 8 \times C_t^{fac}$ meters for PL1, $C_2^{max} = 18 \times C_t^{fac}$ meters for PL2 and $C_3^{max} = 5 \times C_t^{fac}$ meters for CM.

The warehouse can store up to a maximum of $s^{max} = 7200$ meters of end-product at any given time. It is assumed that there is sufficient WIP capacity in front of PL1 to meet the total demand for all products, *i.e.*, $u_1^{max} = \sum_{i\in\mathcal{I},t\in\mathcal{T}} d_{it}$. It is possible to transport at most $u_2^{max} = 5400$ meters of WIP inventory from the warehouse to feed PL2. In front of CM, the available space can store at most $u_3^{max} = 3600$ meters.

According to the historical sales data, the total demand for all types of felt products is in the range $[100000, 140000]$ meters/month. Moreover, the data indicate that 16% of the total demand is for product−1, 4% is for product−2, 64% is for product−3 and 16% is for product−4.

In the existing production policy of the job shop, the machine speeds are fixed without alteration. PL1 operates at a speed of $v_{1t} = 8$ meter/min, PL2 runs in $v_{2t} = 15$ meter/min, and CM has a speed of $v_{3t} = 5$ meter/min. Given that machine m is operational in period t with the plant capacity of $C_t^{fac} = 720$ mins. Then, the *cycle time* $\sigma_{mt} = C_{mt} / v_{mt}$ mins represent the operating duration of machine m in period t. Thus, the *utilization* of machine m in period t is calculated as $\gamma_{mt} = \sigma_{mt} / C_t^{fac} \in [0,1]$.

To meet the product demands, the company applies a production order convention as product−3, product−1, product−4, and product−2. That is, when there are demands for both product−3 and product−4 on PL1 in a period, PL1 will first process product−3 and complete its production before switching to product−4.

4.2. The MSO Model for the Felt Company

The felt company, as described in Section 4.1, has specific constraints in the MSO model as given by constraints (16)−(19).

$$u_{1mt} = u_{3mt} = 0 \quad \forall m \in \boldsymbol{M}, \mathrm{t} \in \boldsymbol{T} \tag{16}$$

$$C_{1t} \leq 8 \times C_t^{fac} \mathrm{x}_{1\mathrm{t}} \quad \forall \mathrm{t} \in \boldsymbol{T} \tag{17}$$

$$C_{2t} \leq 18 \times C_t^{fac} \mathrm{x}_{2\mathrm{t}} \quad \forall \mathrm{t} \in \boldsymbol{T} \tag{18}$$

$$C_{3t} \leq 5 \times C_t^{fac} \mathrm{x}_{3\mathrm{t}} \quad \forall \mathrm{t} \in \boldsymbol{T} \tag{19}$$

In particular, the company does not hold any WIP inventory for cylinder products $i \in \{1,3\}$, as stated in constraints (16). The production requirements and machine capabilities set the upper bounds for the speeds of PL1, PL2, and CM as defined in constraints (17), (18) and (19), respectively.

The aim of the MSO model is to automate the decisions regarding machine speeds and production schedules while minimizing the total production cost. By determining the optimal machine speeds, the MSO model reduces unnecessary WIP and end-item inventory, resulting in improved production costs compared to the company's existing practices. After embedding constraints (16)−(19) into the MSO model presented in Section 3, we analyzed the execution of the felt manufacturing system, as detailed in the following section.

5. Computational Results

In this section, we use the data from the felt manufacturing system described in Section 4.1 and the adapted MSO model from Section 4.2 to conduct computational experiments. We present an optimized machine speed control system for the felt company based on the results obtained from the MSO model.

All experiments are conducted online using Google Colab, with Python programming language and the Pyomo package, utilizing GNU Linear Programming Kit (GNPK) optimization solver. The computational parameters are summarized in Table 3.

Table 3. List of Computational Parameters

(I, M)	$(4, 3)$
T	5 periods
C_t^{fac}	720 mins
Total Demand	$\text{U}[D_l, D_u] = [100K, 140K]$
s^{max}	7200 meters
(u_2^{max}, u_3^{max})	$(5400, 3600)$ meters
r_{mt}	$(1.16, 3.09, 0)$ TL/meter/period
f_{mt}	(20.83, 4.17, 20.83) TL/period
p_{it}	$(6, 8.5, 19.5, 22)$ TL/meter
h_{it}	$(0.75, 0.375, 0.75, 0.375)$ TL/meter
w_{imt}	$(1.25, 1.25, 1.25, 1.25)$ TL/meter for $m = 1$ $(0, 0, 11.4, 11.4)$ for $m = 2$ $(0, 1.25, 0, 1.25)$ for $m = 3$

We randomly generate a total demand within the range $\text{U}[D_l, D_u]$ and distribute it across the planning horizon to determine the individual product demands, based on the historical demand pattern described in Section 4.1. We illustrate the existing production system of the felt company over $T = 5$ periods in Section 5.1. Then, in Section 5.2, we demonstrate the automated production planning and machine speed control through the MSO model for $T = 5$ periods. In Section 5.3, we discuss the enhancements achieved by the MSO production system compared to the current production system. Additionally, we perform experiments on the MSO model under varying demand scenarios and planning horizon parameters to analyze its computational complexity experimentally.

5.1. The Current Felt Production System

In this section, we consider an instance with a planning horizon $T = 5$ periods and describe the existing production planning in the felt company.

We randomly generate the product demands for each period t according to the demand pattern (16%, 4%, 64%, 16%) for $i \in \mathcal{I}$, respectively. The product demands for each period are provided in Table 4. As expected, the highest demand is for product−3, while the lowest demand is for product−2.

We assume that the facility operates with a daily capacity C_t^{fac}= 720 mins. Therefore, the maximum production capacity of PL1 is $C_1^{max} = 8 \times C_t^{fac} = 5760$ meters, PL2 is $C_2^{max} = 18 \times C_t^{fac} = 12960$ meters, and CM is $C_3^{max} = 5 \times C_t^{fac} = 3600$ meters.

Table 4. The Product Demands

d_{it}	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$
$i = 1$	951	943	969	967	931
$i = 2$	242	240	235	227	236
$i = 3$	3789	3841	3791	3794	3817
$i = 4$	940	956	953	966	934

In the current production system, all machines operate at a constant speed throughout the entire period. Note that, PL1 and PL2 have the capability to adjust their speeds, but this feature is not utilized by the company at present. In the existing system, the machine speeds are fixed to $v_{1\text{t}} = 8$ meters/min for PL1, $v_{2\text{t}} = 15$ meters/min for PL2, and $v_{3\text{t}} = 5$ meters/min for CM. Each machine runs at its fixed speed to complete the assigned production quantity and remains idle for the remainder of the period.

There is a limited storage area in front of PL1 and CM, while there is no space in front of PL2. Therefore, when the output from PL1 is insufficient to meet the scheduled production needs of PL2, the WIP products are retrieved from the warehouse and fed to PL2 by an operator. Additionally, if a machine is assigned to produce multiple products in a period, the production sequence follows a priority order, which is given as product−3, product−1, product−4, and product−2.

There is no end-item inventory maintained in the system, *i.e.*, $s_{it} = 0$ for all $i \in \mathcal{I}$, according to the existing production system simulation results. Table 5 summarizes the WIP inventory levels across the planning horizon. We assume that there is sufficient WIP stored in front of PL1 for each product. For instance, the initial WIP inventory for product−1 at PL1 is equal to its total demand, *i.e.*, $u_{110} = d_{11} + \cdots + d_{15} = 951 + \cdots + 931 = 4761$. This initial WIP inventory is consumed throughout the planning horizon as production progresses in PL1.

The production route for product−2 begins with PL1 and proceeds to CM. Since the output from PL1 in period−1 becomes available for CM in period−2, $u_{230} = 806$ meters of WIP are retrieved from the warehouse to meet the period−1 demand for product−2. Note that, this amount is higher than the demand $d_{21} = 242$ due to the machine capacity restrictions and the product priority order.

PL1 is the first machine for product−3, and its output will be available for PL2 in the following period. Then, PL2 initially uses the WIP inventory of $u_{320} = 3789$ meters of WIP inventory from the warehouse to fulfill the demand $d_{31} = 3789$ in period−1. Similarly, the output of PL1 for product−4 will be available for PL2 in the next period, and for CM in the period after that. To account for the time gap, CM stores $u_{430} = 1896$ meters and $u_{431} = 956$ meters of WIP inventory, considering both the machine capacities and product priorities. As a result, the total amount of WIP inventory used by PL2 and CM from the warehouse is 8514 meters for the current production system.

Table 5. The WIP Inventory Levels (meters) in the Current Production System

u_{imt}	$t = 0$	1	2	3	4	5
u_{11t}	4761	3810	2867	1898	931	0
u_{21t}	1180	1165	1105	1042	806	806
u_{23t}	806	564	339	164	0	0
u_{31t}	19032	15191	11400	7606	3789	0
u_{32t}	3789	0	0	0	0	0
u_{41t}	4749	3796	2830	1896	1156	116
u_{42t}	0	0	0	0	0	0
u_{43t}	1896	956	0	0	0	0

The machine production schedule is given in Table 6. PL1 is responsible for producing all product types following the priority rule. Hence, the production quantities for product−3 and product−1, *i.e.*, y_{31t} and y_{11t}, meet the corresponding demands at each period. PL1 has a capacity of 5760 meters per period, and the remaining capacity is allocated to product−4 and then product−2. However, due to capacity limit of PL1, the production of product−2 falls short of the demand. As a result, the WIP deficit for product−2 in front of CM is supplemented from the warehouse, *i.e.*, $u_{230} = 806$ meters.

Table 6. The Production Quantities (meters) in the Current Production System

y_{imt}	$t = 0$	1	2	3	4	5
y_{11t}	0	951	943	969	967	931
y_{21t}	0	15	60	63	236	0
y_{23t}	0	242	240	235	227	236
y_{31t}	0	3841	3791	3794	3817	3789
y_{32t}	0	3789	3841	3791	3794	3817
y_{41t}	0	953	966	934	740	1040
y_{42t}	0	0	953	966	934	740
y_{43t}	0	940	956	953	966	934

Due to the time lag in the machine production schedules, PL2 utilizes the warehouse WIP inventory of product−3 in period−1, *i.e.*, $y_{321} = u_{320} = 3789$ meters. Similarly, CM consumes the warehouse WIP for product−4 in the first two periods. To replenish the warehouse WIP inventory for future production needs, PL1 continues with the production of product−3 in period−5, product−4 in period−4 and 5, *i.e.*, $y_{315} = 3789$, $y_{414} = 740$, and $y_{415} = 1040$. Additionally, PL2 continues producing product−4 in period−5, *i.e.*, $y_{425} = 740$.

The production level of a machine represents the total production in a period, as shown in Table 7. For instance, the production level of machine−3 in period−1 is calculated as $C_{31} = y_{231} + y_{431} = 242 + 940 = 1182$ meters. From the results, we observe that PL1 operates at its maximum capacity of 5760 meters in all periods. The product demands and their corresponding production routes suggest that PL1 experiences the highest load, followed by PL2 and CM has the lowest load. All machines are in operation during every period, *i.e.*, $x_{mt} = 1$ for all $m \in \mathcal{M}, t \in \mathcal{T}$.

Table 7. The Production Levels of Machines (meters) in the Current Production System

C_{mt}	$t = 0$	1	2	3	4	5
C_{1t}	0	5760	5760	5760	5760	5760
C_{2t}	0	3789	4794	4757	4728	4457
C_{3t}	0	1182	1196	1188	1193	1170

The facility operates for C_t^{fac}= 720 mins per day. Then, the machine cycle time can be calculated as $\sigma_{mt} = C_{mt} / v_{mt}$ mins. Here, v_{mt} represents the constant machine speeds in the current production system, *i.e.*, 8, 15, and 5 meters/min for PL1, PL2, and CM, respectively. Table 8 reports that PL1 has the highest cycle time, followed by PL2 and CM has the lowest cycle time.

Table 8. The Machine Cycle Times (mins) in the Current Production System

σ_{mt}	$t = 0$	1	2	3	4	5
σ_{1t}	0	720.0	720.0	720.0	720.0	720.0
σ_{2t}	0	252.6	319.6	317.2	315.2	303.8
σ_{3t}	0	236.4	239.2	237.6	238.6	234.0

We can obtain the machine utilizations as $\gamma_{mt} = \sigma_{mt} / C_t^{fac} \in [0,1]$ based on the cycle times. Table 9 shows that PL1 operates at 100% utilization, PL2 runs at an average utilization of 42%, and CM operates with constant utilization of 33%.

Table 9. The Machine Utilizations in the Current Production System

γ_{mt}	$t = 0$	1	2	3	4	5
γ_{1t}	0	1.00	1.00	1.00	1.00	1.00
γ_{2t}	0	0.35	0.44	0.44	0.44	0.42
γ_{3t}	0	0.33	0.33	0.33	0.33	0.33

The overall production cost includes machine setup costs, production costs, WIP and end-item inventory holding costs, and machine speed costs, as formulated in the objective (1). Then, the overall production cost for the current production system can be found as 781,453 TL for $T = 5$ days.

The company aims to reduce the overall production cost by minimizing the WIP and end-item inventory in the system through dynamic machine speed adjustments. To achieve this, the company plans to implement the MSO model to automate production planning and machine speed decisions throughout the planning horizon.

5.2. The MSO Production System

In this section, we revisit the same instance introduced in Section 5.1 and demonstrate the results of the MSO model for controlling the production system.

The MSO model keeps end-item inventory only for product−4, *i.e.*, $s_{41} = 1520$, $s_{42} = 564$, $s_{43} = 339$, $s_{44} = 164$, $s_{45} = 0$, and $s_{\text{it}} = 0$ for $i \in \{1, 2, 3\}$. The MSO model ultimately aims to satisfy the product demands primarily from the production of PL1, rather than relying on the warehouse WIP inventory, as shown in Table 10.

Table 10. The WIP Inventory Levels (meters) in the MSO Production System

u_{imt}	$t = 0$	1	2	3	4	5
u_{11t}	4761	3810	2867	1898	931	0
u_{21t}	1180	940	705	478	242	242
u_{23t}	242	0	0	0	0	0
u_{31t}	19032	15191	11400	7606	3789	3789
u_{32t}	3789	0	0	0	0	0
u_{41t}	4749	4021	3230	2460	2460	2460
u_{42t}	0	0	0	0	0	0
u_{43t}	2460	0	0	0	0	0

However, due to the machine time lags in the production sequence, the MSO model also utilized some warehouse WIP inventory, *i.e.*, $u_{230} = 242$, $u_{320} = 3789$, and $u_{430} = 2460$. We observe that PL2 and CM continued production without requiring additional warehouse WIP once the outputs of PL1 became available. Consequently, the total warehouse WIP inventory for PL2 and CM was reduced to 6491 meters which represents a 24% decrease compared to the current production system.

We observe that all machines operate during all periods, *i.e.*, $x_{mt} = 1$ for all $m \in \mathcal{M}, t \in \mathcal{T}$. The MSO model generates the production schedule without adhering to the product priority order seen in the current system, as shown in Table 11. This approach enables optimal production level allocation across the machines, effectively minimizing WIP inventory in the system. Compared to the current production system, the MSO model directs CM to produce for the end-item inventory of product−4, as it is more cost-effective than maintaining WIP inventory, *i.e.*, $h_{it} \leq w_{imt}$ for all $i \in \mathcal{I}$**.** This adjustment allows PL1 to efficiently supply product−2 to CM within capacity limits, eliminating the warehouse WIP inventory for product−2, *i.e.*, $u_{23\text{t}} = 0$ for $t > 0$. Unlike the current production system, the MSO model focuses solely on meeting immediate product demand and avoids building up warehouse WIP for future production, *i.e.*, $y_{315} = \text{y}_{414} = \text{y}_{415} = \text{y}_{425} = 0$. This dynamic approach results in more streamlined operations and reduced inventory costs.

Table 11. The Production Quantities (meters) in the MSO Production System

y_{imt}	$t = 0$	1	2	3	4	5
y_{11t}	0	951	943	969	967	931
y_{21t}	0	240	235	227	236	0
y_{23t}	0	242	240	235	227	236
y_{31t}	0	3841	3791	3794	3817	0
y_{32t}	0	3789	3841	3791	3794	3817
y_{41t}	0	728	791	770	0	0
y_{42t}	0	0	728	791	770	0
y_{43t}	0	2460	0	728	791	770

Table 12 summarizes the total production levels of PL1, PL2 and CM at each period. We note that PL1 operates at its maximum capacity 5760 meters during the first three periods. As product demands are gradually met, the production level of PL1 decreases over time. Meanwhile, PL2 reveals a relatively stable production output, averaging 4264 meters per period.

Conversely, CM operates at its highest capacity in the first period to meet the demand of product−2 and to store the end-item inventory of product−4 using the warehouse WIP inventory. This production plan overcomes the time lag in the machine production schedules and the capacity constraints of PL1. For $t > 0$, the production system is driven by the outputs of PL1, eliminating the need for warehouse WIP inventory for PL2 and CM.

Table 12. The Production Levels of Machines (meters) in the MSO Production System

C_{mt}	$t = 0$	1	2	3	4	5
C_{1t}	0	5760	5760	5760	5020	931
C_{2t}	0	3789	4569	4582	4564	3817
C_{3t}	0	2702	240	963	1018	1006

The cycle time of machine m in period t is calculated as $\sigma_{mt} = \min\left\{\frac{C_{mt}}{v_m^{min}}, C_t^{fac}\right\}$ mins, where v_m^{min} is the minimum speed of machine m. Table 13 reports the machine cycle times based on the capacities C_{mt} in the MSO production system.

For instance, machine−1 in period−4 has a cycle time $\sigma_{14} = \min\left\{\frac{C_{14}}{v_1^{min}}, C_t^{fac}\right\} = \min\left\{\frac{5020}{5}, 720\right\} = 720$ mins. This means, PL1 should operate throughout the entire period at a speed higher than the minimum $v_1^{min} = 5$ meters/min.

Similarly, for machine−1 in period−5 we calculate the cycle time as $\sigma_{15} = \min\left\{\frac{C_{15}}{v_1^{min}}, C_t^{fac}\right\} = \min\left\{\frac{931}{5}, 720\right\} =$ 186.2 mins. That is, PL1 can complete the production for period−5 in 186.2 mins while operating at the minimum speed $v_1^{min} = 5$ meters/min.

Table 13. The Machine Cycle Times (mins) in the MSO Production System

σ_{mt}	$t = 0$	1	2	3	4	5
σ_{1t}	0	720.0	720.0	720.0	720.0	186.2
σ_{2t}	0	252.6	304.6	305.3	304.3	254.5
σ_{3t}	0	540.4	48.0	192.6	203.6	201.2

The machine cycle time σ_{mt} determines the speed of machine m as follows:

$$v_{mt} = \frac{C_{mt}}{C_t^{fac}} \quad \text{if} \quad \sigma_{mt} = C_t^{fac} \tag{20}$$

$$v_{mt} = v_m^{min} \quad \text{if} \quad \sigma_{mt} < C_t^{fac} \tag{21}$$

Table 14 provides the machine execution speeds based on Equations (20) and (21). According to the MSO model results, PL2 and CM maintain constant speeds of 15 and 5 meters/min, respectively, throughout all periods. On the other hand, PL1 adjusts its speed dynamically in response to varying product demands. By combining the data from Tables 13 and 14, the operational details of machine m in period t can be determined, showing it operates for σ_{mt} mins at speed v_{mt}. For instance, PL1 operates at a speed of 5 meters/min for 186.2 mins in period−5, and CM runs for 48 mins at a constant speed of 5 meters/min in period−2.

Table 14. The Machine Speeds (meter/min) in the MSO Production System

v_{mt}	$t = 0$	1	2	3	4	5
v_{1t}	0	8	8	8	7	5
v_{2t}	0	15	15	15	15	15
v_{3t}	0	5	5	5	5	5

Table 15 presents the machine utilizations, calculated as $\gamma_{mt} = \sigma_{mt} / C_t^{fac} \in [0,1]$, based on the cycle times. The machine utilizations fluctuate in alignment with the cycle times across the planning horizon. According to the results, the average utilization rates are 85% for PL1, 39% for PL2, and 33% for CM. Compared to the current production system described in Section 5.1, both PL1 and PL2 operate at lower utilization levels in the MSO model. This reduction is attributed to the optimized production strategy, which minimizes the reliance on warehouse WIP inventory. In contrast, the current system maintains higher utilization for PL1 and PL2 due to its focus on building up WIP inventory to support future production plans.

Table 15. The Machine Utilizations in the MSO Production System

γ_{mt}	$t = 0$	1	2	3	4	5
γ_{1t}	0	1.00	1.00	1.00	1.00	0.26
γ_{2t}	0	0.35	0.42	0.42	0.42	0.35
γ_{3t}	0	0.75	0.07	0.27	0.28	0.28

In the end, the production schedule of the MSO model has an overall production cost of 777,600 TL including the production costs, WIP and end-item inventory holding costs, and the costs associated with altering machine speeds.

5.3. Comparison of the Current and MSO Production Systems

When comparing the results presented in Tables 5-9 for the current production system, and Tables 10-15 for the MSO production system, we observe a significant improvement over a planning horizon of $T = 5$ days. Firstly, the MSO model achieves a cost reduction of 3853 TL, bringing the total cost down to 777,600 TL. This reduction is largely attributed to the optimization of machine speeds, which minimizes WIP inventory, reduces holding costs, and eliminates the need for excessive warehouse storage.

In terms of production strategy, the current system relies on building WIP inventory for future production, leading to higher machine utilizations but increased inventory holding costs. As a result, the total warehouse WIP inventory used is 8514 meters in the current system.

On the other side, the MSO model implements dynamic scheduling and machine speed control, aligning production closely with demand and minimizing unnecessary inventory buildup. Therefore, the MSO model reduces warehouse WIP inventory by 24% to 6491 meters, prioritizing direct production to meet demands.

When comparing the machine utilizations, PL1 operates at 100% utilization at its highest speed throughout the planning horizon in the current system. Since PL1 is the first machine in the production sequence for all products, its utilization determines the throughput capacity of the entire production system. As a result, the throughput capacity cannot be increased in the current system because PL1 is fully utilized throughout the planning horizon.

In contrast, the MSO model optimizes the machine speeds across the system, successfully reducing the utilization of PL1 from 100% to 85%, and PL2 from 42% to 39%. The utilization of CM remains at 33%, as it is not designed for speed adjustments. With the available capacity in PL1, the throughput of the system can be increased without the need for additional investments in new machines or workers. This optimization approach not only increases efficiency but also enhances the overall production capacity within the existing system.

We further experimented with the MSO model to assess potential improvements on the current system. In the case presented in Sections 5.1 and 5.2, the throughput, or total demand for all products, was 35669 meters. By successfully reducing the utilization of PL1, the critical machine in the system, to 85%, we explored different throughput levels. We observed that the throughput can reach a maximum of 39449 meters without causing infeasibility, representing an 11% increase in throughput. At this new throughput level, the machine utilizations are 86% for PL1, 43% for PL2, and 36.4% for CM. This demonstrates that the current system has the capacity to produce 11% more if machine speeds are optimized using the MSO model for a 5-day planning horizon. As a result, the productivity and profitability of the company can be enhanced without the need for additional investments in new machines or work shifts.

In this scenario, we noted that the machines are not fully utilized, indicating that the critical constraints are the limits of the WIP and end-item inventory capacities. Therefore, the throughput of the system could be further improved if these inventory capacities are expanded.

In our final experiment, we run the MSO model with a one-hour time limit for various planning horizons $T \in \{20, \dots, 180\}$ and different demand parameters to investigate its computational complexity experimentally. Based on our earlier experiments, we test two demand levels: the current level of 35669 meters per 5 days, and a higher level of 39449 meters per 5 days. Figure 3 shows the computational complexity of the MSO model graphically.

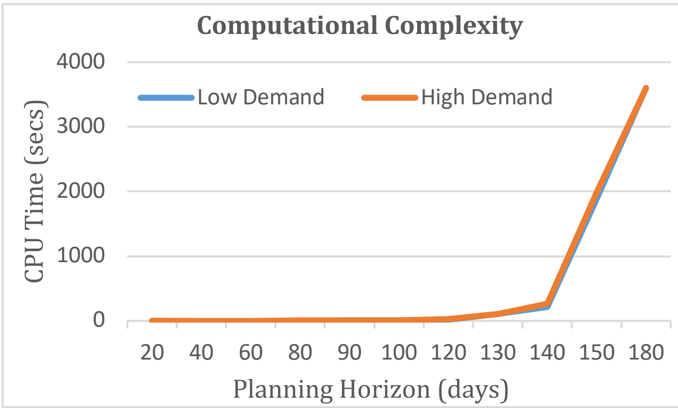

Figure 3. Computational Complexity of the MSO model

The experiments reveal that execution time increases as the planning horizon lengthens, reaching the one-hour time limit at $T = 180$ days. The demand level has a slight impact on computational time, with slightly longer execution times observed for high-demand scenarios. The MSO model can generate an optimal production plan with machine speeds in under 5 mins for $T \leq 140$ days.

Consequently, the MSO model demonstrates a more efficient approach, offering cost savings and improved resource utilization while reducing dependency on warehouse WIP inventory. The complexity of the MSO model is acceptable, as it can generate the optimal production plan and machine speeds within one hour for a six-month planning horizon. This makes the MSO model suitable for practical implementation in the felt company.

6. Conclusions

In this work, we explored an extension of the well-known CLSP specifically designed for a felt production system. Unlike discrete-item manufacturing, felt production involves producing rolls of felt measured in meters. Besides, the felt production includes chemical operations, necessitating machines to operate within specific speed limits to ensure proper processing.

Customer demand can be met without backlogging while eliminating unnecessary WIP and end-product inventory by optimally controlling machine speeds during each period. Implementing such intelligent production control fosters a sustainable production system, as operating machines at lower speeds reduces both energy consumption and machine downtimes.

We addressed a felt production system constrained by machine speed limits and bounded by end-product and WIP inventory levels. To optimize this system, we proposed an MSO model designed to address a deterministic demand pattern within a finite planning horizon. Our MSO model was tailored for a felt manufacturing company operating in Istanbul, Türkiye. For this case study, we conducted computational experiments to demonstrate the effectiveness of automated production system control.

In the current production system, machine speeds remain constant throughout the planning horizon. The machines handle multiple products within their limited capacities. When a machine's capacity is insufficient to meet the multi-product demand in a given period, a predefined priority order of product-3, product-1, product-4, and product-2 is applied. However, this prioritization can lead to deficiencies in WIP inventory during the production sequence, necessitating the use of existing WIP inventory stored in the warehouse to fulfill product demand. This approach increases the requirement for storage space and elevates warehouse inventory holding costs over time.

Alternatively, the MSO model dynamically adjusts machine speeds to ensure a smooth product flow in the production sequence while minimizing WIP inventory. Unlike the current production system, the MSO model prioritizes storing end-item inventory, thereby eliminating the reliance on warehouse WIP inventory. The results presented in Section 5.2 align with this objective, as the warehouse WIP inventory is utilized only at $t = 0$ to initiate production. Throughout the planning horizon, the WIP inventory levels for PL2 and CM remain at zero. As the initial machine in the production sequence, PL1 strategically retains WIP inventory to trigger production across the system.

The experimental setup for the felt manufacturing company includes three machines and four products. Computational analysis shows that, in this setting, the MSO model can generate an optimal six-month production plan with machine speed adjustments in less than an hour. Consequently, the MSO model effectively automates machine speed control and production scheduling for the existing felt production system.

The MSO problem, formulated as a MILP in this study, is NP-Hard. Future research could explore heuristic solution approaches, such as the relax-and-fix heuristic, to obtain near-optimal solutions within a reasonable computation time. Additionally, the MSO problem could be adapted to other continuous production systems, such as float glass manufacturing. Since backlogging is currently not permitted in the MSO model, a potential extension could include a backlogging mechanism to address the lot sizing and scheduling problem more comprehensively.

Contribution of Researchers

In this research, Author 1 contributed to the development of the mathematical model and its adaptation to the case study. Additionally, Author 1 collected data for the case study and conducted the computational experiments.

Conflict of Interest

There is no conflict of interest with any person/institution in this article.

References

Akbalik, A., Penz, B. ve Rapine, C. (2015). Capacitated lot sizing problems with inventory bounds. *Annals of Operations Research*, 229, 1–18. doi: https://doi.org/10.1007/s10479-015-1816-6

Absi, N., Detienne B. ve Dauzere-Peres, S. (2013). Heuristics for the multi-item capacitated lot-sizing problem with lost sales. *Computers & Operations Research*, 40(1), 264–272. doi: https://doi.org/10.1016/j.cor.2012.06.010

Ben Ammar, H., Ayadi O. ve Masmoudi, F. (2020). An effective multi-objective particle swarm optimization for the multi-item capacitated lot-sizing problem with set-up times and backlogging. *Engineering Optimization*, 52(7), 1198-1224. doi: https://doi.org/10.1080/0305215X.2019.1636978

Brahimi, N., Dauzere-Peres, S. ve Najid, N.M. (2005). Capacitated multi-item lot-sizing problems with time windows. *Operations Research*, 54, 951–967. doi: https://doi.org/10.1287/opre.1060.0325

Chung-Yee, L., Çetinkaya, S. ve Wagelmans, A.P.M. (2000). A dynamic lot-sizing model with demand time windows. *Management Science*, 47(10), 1384-1395. doi: https://doi.org/10.1287/mnsc.47.10.1384

Florian, M., Lenstra, J.K. ve Kan, A.H.G.R. (1980). Deterministic production planning: Algorithms and complexity. *Management Science*, 26(7), 669–679. doi: https://doi.org/10.1287/mnsc.26.7.669

Ganesh, K. (2019). *Capacitated lot sizing problems in process industries*. Cham, Switzerland: Springer. doi: https://doi.org/10.1007/978-3-030-01222-9

Geng, Z. ve Yuan, J. (2023). Single-machine scheduling of multiple projects with controllable processing times. *European Journal of Operational Research*, 308(3), 1074-1090. doi: https://doi.org/10.1016/j.ejor.2023.01.026.

Gramani, M., França, P. ve Arenales, M. (2009). A lagrangian relaxation approach to a coupled lot-sizing and cutting stock problem. *International Journal of Production Economics,* 119(2), 219–227. doi: https://doi.org/10.1016/j.ijpe.2009.02.011

Jans, R. ve Degraeve, Z. (2008). Modeling industrial lot sizing problems: A review. I*nternational Journal of Production Research*, 46, 1619–1643. doi: https://doi.org/10.1080/00207540600902262

Karimi, B., Ghomi, S.M.T.F. ve Wilson, J.M. (2003). The capacitated lot sizing problem: a review of models and algorithms. *Omega,* 31, 365-378. doi: https://doi.org/10.1016/S0305-0483(03)00059-8

Kimms, A. (2012). *Multi-level lot sizing and scheduling: methods for capacitated, dynamic, and deterministic models*. Springer Science & Business Media. doi: https://doi.org/10.1007/978-3-642-50162-3

Koca, E., Yaman, H. ve Aktürk, A.S. (2015). Stochastic lot sizing problem with controllable processing times. *Omega,* 53, 1-10. doi: https://doi.org/10.1016/j.omega.2014.11.003

Levin, A. ve Shusterman, T. (2023). Weighted throughput in a single machine preemptive scheduling with continuous controllable processing times. *Acta Informatica*, 60 (2), 101-122. doi: https://doi.org/10.1007/s00236-022-00430-4

Martinez, K.Y., Toso, E.A. ve Morabito, R. (2016). Production planning in the molded pulp packaging industry. *Computers & Industrial Engineering,* 98, 554–566. doi: https://doi.org/10.1016/j.cie.2016.05.024

Masmoudi, O., Yalaoui, A., Ouazene, Y. ve Chehade, H. (2016). Multi-item capacitated lot- sizing problem in a flow-shop system with energy consideration. *8th IFAC Conference on Manufacturing Modelling, Management and Control,* 49(12), 301–306. doi: https://doi.org/10.1016/j.ifacol.2016.07.621

Naeem, M., Dias, D., Tibrewal, R., Chang, P.C. ve Tiwari, M. (2012). Production planning optimization for manufacturing and remanufacturing system in stochastic environment. *Journal of Intelligent Manufacturing*, 24. doi: https://doi.org/10.1007/s10845-011-0619-0

Özdamar, L. ve Bozyel, M.A. (2000). The capacitated lot sizing problem with overtime decisions and setup times. *IIE Transactions,* 32(11), 1043-1057. doi: https://doi.org/10.1023/A:1013733024060

Özyörük, D.A. ve Erol, P. (2000). Parti Büyüklüğü Problemleri ile İlgili Literatür Araştırması. *Süleyman Demirel Üniversitesi İktisadi ve İdari Bilimler Fakültesi Dergisi*, 5 (2), 105-119. Access: https://dergipark.org.tr/tr/download/article-file/195154

Quadt, D. (2004). *Lot-sizing and scheduling for flexible flow lines*. Springer Science & Business Media. doi: https://doi.org/10.1007/978-3-642-17101-7

Ramya, R., Rajendran, C., Ziegler, H., Mohapatra, S. ve Ganesh, K. (2019). *Capacitated lot sizing problems in process industries*. Cham, Switzerland: Springer.

Renna, P. (2023) Controllable processing times with limited resources and energy consumption in flow shops: an assessment by simulation, International *Journal of Management Science and Engineering Management*, doi: https://doi.org/10.1080/17509653.2023.2285774

Shabtay, D. ve Steiner, G. (2007). A survey of scheduling with controllable processing times. *Discrete Applied Mathematics.* 155(3), 1643-1666. doi: https://doi.org/10.1016/j.dam.2007.02.003

Wang, S., Wu, R., Chu, F. ve Yu, J. (2022). Unrelated parallel machine scheduling problem with special controllable processing times and setups. *Computers & Operations Research*, 148, 105990. doi: https://doi.org/10.1016/j.cor.2022.105990

Wagner, H.M. ve Whitin, T.M. (1958). A dynamic version of the economic lot size model. *Management Science*, 5(1): 89–96. doi: https://doi.org/10.1287/mnsc.5.1.89

Wu, T., Shi, L. ve Duffie, N.A. (2010). An hnp-mp approach for the capacitated multi-item lot sizing problem with setup times. *IEEE Transactions on Automation Science and Engineering*, 7, 500–511. doi: 10.1109/TASE.2009.2039134

Yanzhi, L., Yi, T. ve Wang, F. (2012). An effective approach to multi-item capacitated dynamic lot-sizing problems. *International Journal of Production Research*, 50, 5348–5362. doi: https://doi.org/10.1080/00207543.2011.626459